A Framework of hybrid approach for rich VRP studies in terms of Weber basis cost accounting
-Enhanced from three-dimensional case-

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Abstract
Today global and dynamic markets require us to provide cooperative and competitive logistics so that we can build a manufacturing system amenable to sales and operations planning. As a deployment for such practice, keen interests have been paid at logistics optimization known as vehicle routing problem (VRP). Noticing that transportation cost and/or CO$_2$ emission actually depend not only on distance traveled but also weight loaded (Weber basis), in this paper, we first overview our rich VRP studies in a lumped classification. Then, we describe a hybrid meta-heuristic method available for solving commonly various variants in a framework. Moreover, as an emerging and relevant issue to enhance our framework, this study attempts to add another variant associated with 3-dimensional transportation. Finally, through broad comparison of numerical experiments, we examine the solution abilities for solving various real world problems and discuss some prospects on the 3-dimensional transportation.

Key words : Rich VRP, 3-dimensional transportation, Framework of hybrid approach, Weber basis cost accounting.

1. Introduction

Facing with rising markets and various demands on qualified service in competitive transportation system, logistic optimization is becoming a keen interest to provide a sustainable infrastructure aligning to modern societal prospects. As a key technology for such deployment, we have been engaged in the practical studies to resolve vehicle routing problems (VRP) noticing that the transportation cost and/or CO$_2$ emission actually depend not only distance but also loading weight. This cost accounting is known as Weber model and has been applied popularly in a strategic planning like allocation/location problems. Based on this cost accounting, we have developed a hybrid approach that can cope with rich VRP studies efficiently as well as practically.

Overviewing such approaches comprehensively, in this paper, we add a variant so that we can complete a framework of our VRP studies. Actually, we note an emerging interest associated with future logistics systems, i.e., 3-dimensional transportation. Then, to validate the effectiveness of the method, we carry out a numerical experiment.

The rest of the paper is organized as follows. In Section 2, we describe broad problem statement in terms of lumped classification. Then we outline the proposed solution procedure in a hybrid manner in Section 3. Numerical experiments are provided in Section 4. Finally, we give conclusions.

2. Problem statement in terms of lumped classification of VRP

A logistic network problem known as VRP is a combinatorial optimization problem on minimizing the total distance traveled by a fleet of vehicles under various constraints. This transportation of goods from depots to all customers must be considered under the condition that each vehicle must take a circular route with the depot as its starting point and destination as well. Due to NP-hardness, it is almost impossible to solve real world problems through any mathematical programming methods. In contrast, by virtue of amazing progress of meta-heuristic methods, we can
cope with such situation if only near optimal solution is satisfactory.

Besides the later discussions along with a lumped classification depicted in Fig.1, sophisticated deployments have been known in this area. For example, in addition to generic customer demand satisfaction and vehicle payload limit conditions, including some practical concerns such as customer availability or time windows (Mester, Braysy & Dullaert, 2007) and split and mixed delivery (Mota, Campos & Corberan, 2007) are very popular. These extensions are considered both separately and in combined manners (Zhong & Cole, 2005).

Fig. 1 Lumped classification of VRP and associated studies; Top node of each tree denotes the overall property and right hand side figure classifies closely related studies to VRP (Also refer to Fig.2).

2.1 Conventional and Weber basis transportation cost accounting

From our daily experience through driving a car, we know fuel consumption (cost) and CO₂ emission actually depend not only on travelling distance but also loading weight (tonnage-kilo meter basis or Weber basis). In car industries, therefore, reducing weight of vehicle body has been a keen engineering interest all over the world. It means we should use Weber basis to evaluate those instead of the conventional non-Weber basis, i.e., kilo-meter basis. To the best of our knowledge, however, such idea has never been used in the previous VRP studies.

In contrast, this Weber basis has been popularly applied to a formulation of facility location problem. Viewing this class of problems as a decision at strategic level, VRP and location-routing problem may belong to an operational and tactical level, respectively. As pointed out in the above, the Weber basis has been popularly applied to the location problems while not to VRP. Hence, if we apply this basis in VRP, we can evaluate the transportation costs on the same basis both for location and routing problems. To solve the location-routing problem (Shimizu & Fatrias, 2013), therefore, we know it quite relevant (Refer to Fig.2). This is another significance to use the Weber basis for VRP.

Fig. 2 Smooth integration of strategic and operational models for tactical level model; By using Weber basis for VRP, we can evaluate the transportation costs on the same basis both for location and routing problems

The Weber basis is a bilinear model of distance and weight, i.e., \( (\text{Distance})^\gamma \cdot (\text{Weight})^\beta \). A power model of those two quantities is known as the generalized Weber basis and able to give a more practical cost accounting (Watanabe, 2010). It is described as \( \gamma (\text{distance})^\alpha \cdot (\text{weight})^\beta \) where \( \gamma \) denotes a constant and \( \alpha \) and \( \beta \) elastic coefficients for the distance.
and weight, respectively. It should be noted mathematical formulation on the conventional kilo-meter basis refer to a linear problem while those turn to non-linear MIPs that are several times difficult to solve. This fact might greatly amplify the significance of the proposed hybrid meth-heuristic approaches since they can attain at a near optimal solution with an acceptable computational effort.

2.2 Single-depot and multi-depot networks

When we consider only one depot in logistic network, the problem is called single-depot problem or simply VRP. Meanwhile, it is called multi-depot VRP when we consider multiple depots (Refer to Fig.3). Multi-depot VRP is viewed as a variant of location-routing problem since it involves another decision how to allocate the client customers for each depot. Solving allocation problems has a property of combination while routing that of permutation. This fact gives a major cause of high NP-hardness in solution. Due to such integrated difficulty in solution, only a limited number of studies have been done. Regarding the rapidly growing interests on this topic, we can consult the recent review (Montoya-Torre, et al., 2015). From a practical point of view, however, those studies are quite insufficient. Being different from our approach (Shimizu & Sakaguchi., 2014), they basically solved only small benchmark problems with non-Weber basis cost accounting to validate the effectiveness of the proposed methods.

In this class, it is interesting to confirm the effect of decentralization or replacing single-depot (centralized) logistics with multi-depot (decentralized) logistics. In the centralized configuration, we can expect more organized management than the decentralized besides the scale merit. On the other hand, it is possible to take some adaptive management and more relevant pairs between each depot and its client customers in the decentralized logistics. Noting these features, a gain given by \((\text{Cost(Single)} - \text{Cost(Multi)})/\text{Cost(Single)}\) is evaluated (Shimizu, Sakaguchi & Yoo, 2016). Here, \text{Cost(Multi)} and \text{Cost(Single)} denote the costs of multi-depot and single-depot logistics, respectively.

![Fig.3 Configuration of single-depot (centralized) and multi-depot (decentralized) VRP; All customers belong to single-depot (left) while it is necessary to allocate the client customers for anyone of the multiple depots (right).](image)

2.3 Mono mode and multi-mode transportations

Conventionally, VRP has mainly concerned delivery problems. According to the increasing interests in green logistics, however, pickup type is often considered recently. In contrast to delivery, it collects garbage and/or spent products from the distributed pickup points as commonly seen in reverse logistic. Such pickup problem is further classified into direct and drop-by cases. The former is just the dual of the delivery and the later need to drop by another destination to dump the debris collected over the route before returning the starting depot.

Besides such mono-mode VRP, some researchers have been recently interested in VRP with varying pickup and delivery configurations or multi-mode VRP. This mode seems to be the most practical and suitable way to consider reverse logistics. As a special variant of VRP, many researches are interested in VRP with pickup and delivery demands of every customer from certain aspects. They are classified into three categories known as Delivery with backhauls (VRPB), Mixed Pickup and Delivery (MVRP) and Simultaneous Pickup and Delivery (VRPSPD) (Nagy & Salhi, 2005).

Here, we can cope with VRPB by applying the mono mode cases separately in order of delivery and pickup. VRPSPD turns to MVRP when either of pickup demand or delivery demand is placed at each customer. In this sense, VRPSPD is considered as the most practical and general when we discuss on the cooperative and competitive logistics.
in modern society. This VRPSPD is frequently encountered in the distribution system of bottled drinks, groceries, LPG tanks, laundry service of hotels, the reverse logistics, etc. By the simultaneous delivery and pickup (SPD), we can expect to realize a higher loading ratio of vehicle compared with the mono mode. For example, in such a special case that every delivery and pickup demands are equal and every routing path has the same length, the loading ratio of every vehicle with SPD becomes 100% (Refer to Fig.4).

Though growing concerns are paid on this topic after the first work by Min (1989), everyone concerns with single-depot problems. Due to the high complexity as mentioned already, only a few studies (Gajpal & Abad, 2013, Karaoglan et al., 2012) have been known for multi-depot VRPSPD. Likewise, they solved only small problems by heuristic approaches under the non-Weber basis to validate the effectiveness of the proposed methods.

Besides the higher loading ratio of vehicle mentioned above, it is meaningful to ascertain the merit of SPD from the saving rate against the separate transportations, i.e., independent delivery and pickup transportations. It is evaluated by (Cost(P&D) - Cost(SPD))/Cost(P&D). Here, Cost(SPD) and Cost(P&D) denote the costs of SPD and the separate case, respectively (Shimizu, Sakaguchi & Yoo, 2016).

![Fig.4 Comparison of loading capacity among different modes; A vehicle by SPD transports full load in every path, i.e., 2 [ton]. So it comes to 6 [ton] totally while mono mode transportation stays only its half, i.e., 3 [ton]](image)

### 2.4 Single-objective and multi-objective evaluations

Due to diversified ways of thinking, modern logistics often faces with problem-solving that is difficult to resolve simply. In fact, we should notice manifold objectives realizing environmentally benign (low carbon), elaborate (agile service) and/or confidential (low risk) achievements. Generally speaking, there exists a trade-off between anyone of those objectives and the economy (cost). In such situation, therefore, multi-objective optimization is more suitable to cope with VRP (Jozefowicz, Semet & Talbi, 2008).

Under such understanding, we also concerned with a multi-objective location-routing problem. In the first study (Shimizu, Sakaguchi, & Tsuchiya, 2013), we are interested in a multi-objective analysis between cost and service. It should be noticed those objectives are incommensurable with each other and there exists no proper converter to combine them. Then, associated with the available interval of customer or time window, we measured a gap off the window or waiting time as a quality of service. Actually, we draw a Pareto front between the cost and waiting time by using conventional ε-constraint method, and discussed on some properties how to make a final decision through trade-off analysis.

Regarding CO₂ emission, transportation sector occupies a rather higher portion among all sectors of society. Aiming at low carbon logistics is essential for us to realize sustainable society. Hence, it is natural to turn our interest to the environmental issues (Shimizu & Sakaguchi., 2013, Shimizu, Sakaguchi & Shimada, 2015). In its problem formulation, we describe the first objective function that represents the total transportation cost by \( f_1(X) \) [cost unit]. Here, \( X \) denotes the decision variables appeared in the formulation. Then, just replacing the coefficients of the cost factors in \( f_1(X) \) with those of CO₂ emission, we can give the second objective function \( f_2(X) \) [amount of CO₂] to evaluate the CO₂ emission and compute its value in terms of the improved tonnage kilo-meter basis recommended to use by Japanese government (URL, 2015). Moreover, as a promising way to integrate these two functions with different units into one, we introduced a coefficient \( \lambda \) [cost unit/unit amount of CO₂] known as the emission trading rate on CO₂. Then, we can transform the amount of CO₂ emission into the cost and finally formulate a minimization problem with the objective function given by \( f_1(X) + \lambda f_2(X) \). Here, we emphasize again Weber basis is known to be amenable to evaluate both functions in practice.
2.5 Two-dimensional and three-dimensional transportations

Without stepping into a near future delivery system using drone, we should consider a 3-dimensional option to evaluate the cost more correctly. For this purpose, we are now ready for retrieving such geographic data through Elevation API of Google map, for example. In fact, we know vehicle uses more fuel when driving the upward route while less for the downward compared with the flat. Factor \( G(\theta) \) is defined as a scale to adjust such driving condition depending on the difference of elevation between two site \( i \) and \( j \), \( h_{ij} \). Actually, it is given as the value of \( \theta = \tan \theta_{ij} = h_{ij}/d_{ij} \). To get the revised cost \( \text{Cost}(3D) \), this factor is multiplied with the usual 2-dimensional one, \( \text{Cost}(2D) \) as \( \text{Cost}(3D) = G(\theta_{ij}) \cdot \text{Cost}(2D) \).

Since we could not find an appropriate mathematical model to work with this idea, we assume a sigmoid function referring to the literature that discussed on this issue (Takeuchi, 2009), i.e., \( a + b/(1 + e^{-c}) \). The profile described in Fig.5 is reasonable since it suites to our empirical knowledge such that: the factor exceeds 1 for the upward and falls below 1 for the downward; upward band is greater than that of downward; and the factor will be saturated at both edges of range \( \theta \). To evaluate this effect practically in real world applications, we divide the route into sub-routes according to the feature such as upward, flat and downward, respectively. Then we compute the revised distance for 3-dimensional case by Eq.(1).

\[
\tilde{G}(\theta_{ij})d_{ij} = \sum_{(m,n)} \delta_{mn}G(\theta_{mn})
\]

(Eq.1)

where \( \delta_{mn} \) denotes a distance of sub-route involved in route \( i \rightarrow j \). Hence, distance \( d_{ij} \) is given by \( \sum_{(m,n)} \delta_{mn} = d_{ij} \) (See Fig.6 also). Finally, we use this value as the premium or discount factor to compute the transportation cost between two sites.

![Fig. 5 A scheme of adjusting factor with slope angle; Factor increases over 1 for the upward and decreases below 1 before saturation at both ends of \( \theta \)](image)

![Fig. 6 A scheme to compute the revised distance for complex configuration of slope: Dividing the route from \( i \) to \( j \) into sub-routes, we obtain the factor \( G \) as the weighted average of each factor with sub-distances \( \delta_{ij} \).](image)

To examine the effect of this factor intuitively, we carry out a simulation in terms of a simple pattern, i.e., just up and down movements. We obtain a contour map of \( G \) on the domain of elevation of peak and its location in Fig.7. Generally speaking, the factor becomes greater as slope become steeper and up-distance longer and vice versa. However, since such feature is not straightforwardly applicable, by virtue of elevation date from a certain digital map, it is meaningful to evaluate the reality from the above idea, i.e., Eq.(1). To evaluate such concern, we give an index called gap that is given by \( (\text{Cost}(3D(2D^*))) - \text{Cost}(3D^*)/\text{Cost}(3D^*) \) where \( \text{Cost}(3D^*) \) and \( \text{Cost}(3D(2D^*)) \) denote the transportation costs in terms of the optimal 3-dimensional solution and the transformed one using the conventional optimal 2-dimensional solution, respectively. It represents a gain that we can return if we evaluate the costs through the
3-dimensional basis instead of the usual 2-dimensional one in real 3-dimensional environment.

![Contour map with properties of slope.](image)

Fig. 7 A contour map with properties of slope.; For a simple pattern like just up and down, the factor becomes greater as slope become steeper and up-distance longer and vice versa.

3. Hierarchical hybrid approach for practical solution

![Flowchart of the proposed approach](image)

Fig. 8 Flowchart of the proposed approach; Single depot problem is worked with Weber basis saving method and modified tabu search within the inner loop search. Moreover, for multi-depot problem, graph algorithm to solve MCF problem is added and search is carried out within the outer-loop.

Any of the above problems formulated mathematically belong to an NP-hard class and become almost impossible to obtain an exact optimal solution for real-world problems. Hence, instead of any commercial solvers, it is meaningful to provide a practical method that can derive a near optimum solution with an acceptable computational effort. For this purpose, we can apply our hierarchical hybrid approach in the same manner to everyone mentioned so far and more in advance. Thereat, we use three major components, i.e., graph algorithm to solve MCF problem, Weber basis saving method and modified tabu search. The graph algorithm is used to allocate the client customers for each depot, Weber basis saving method to derive an initial solution of VRP in the inner loop search and the modified tabu search to improve the tentative solution both in the inner and outer loop searches, respectively. Hence, the allocation problem and outer loop search are to be skipped for single depot problems.

In Fig. 8, we give a flowchart of the proposed procedure. Actually, in terms of this approach, we might cope with every variant just by replacing the saving value used in the Weber basis saving method with the relevant one for the
problem under consideration. Moreover, this has a great potential to cope with the others that might be derived from the combination of child nodes of some trees in Fig.1. Below, major components in the procedure are explained briefly.

### 3.1 Allocation of customers for multi-depot problems

Deciding the client customers to each depot in a suitable manner, we can move on the next step to solve the multiple single-depot problems in turn. This is equivalent to solve the following linear programming problem (LP) for delivery problem.

\[
(p.1) \min \sum_{j \in J} \sum_{k \in K} c_{jk} g_{jk} + \sum_{j \in J} H_j \sum_{k \in K} g_{jk}
\]

subject to

\[
\sum_{k \in K} g_{jk} \leq U_j, \quad \forall j \in J
\]

\[
\sum_{j \in J} g_{jk} = q_k, \quad \forall k \in K
\]

\[
g_{jk} \geq 0, \quad \forall j \in J, \forall j \in J
\]

where \( g_{jk} \) denotes the amount allocated from depot \( k \) to customer \( j \) [ton]; \( c_{ij} \): transportation cost per unit load per unit distance of vehicle \( v \) [cost unit /ton/km]; \( d_{jk} \): path distance between \( j \in J \) and \( k \in K \) [km]; \( H_j \): handling cost of depot \( j \) [cost unit /ton], \( q_k \): delivery demand of customer \( k \) [ton]; \( U_j \): maximum capacity of depot \( j \) [ton]; \( J \): index set of depot; \( K \): index set of customer.

Actually, to enhance the solution ability, we apply the graph algorithm of MCF problem instead of solving the above LP directly. We show this graph and label information on the edge of graph in Fig.9. Here, the depot with no inflow from the source in the MCF graph will not be opened. After all, we can allocate every customer to each depot efficiently and practically as well.

In the cases of pickup or SPD, the above Eq.(3) might be replaced with Eq.(4) or Eq.(5), respectively.

\[
\sum_{j \in J} g_{jk} = p_k, \quad \forall k \in K
\]

\[
\sum_{j \in J} g_{jk} = r_k = \max(q_k, p_k), \quad \forall k \in K
\]

where \( p_k \) denotes pickup demand of customer \( k \).

![Fig.9 MCF graph for allocation for delivery problem](image)

<table>
<thead>
<tr>
<th>Map</th>
<th>( \phi_i ) for Weber</th>
<th>( \phi_j ) for Generalized Weber*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delivery</td>
<td>Pickup</td>
<td>SPD</td>
</tr>
<tr>
<td>2-dimension</td>
<td>( \gamma q_i^{a-1} d_0^{b-1} )</td>
<td>( \gamma p_i^{a-1} d_0^{b-1} )</td>
</tr>
<tr>
<td>3-dimension</td>
<td>( G(z_0) )</td>
<td>( G(z_0) \gamma q_i^{a-1} d_0^{b-1} )</td>
</tr>
</tbody>
</table>

*Approximated by apparent linearization

This approach is suitable compared with the other methods such as Voronoi diagram (Man et al., 2012), cluster divisions (Esnaf & Küçükdeniz, 2009), polar angles between the depot and the customers (Gillett & Miller, 1974) etc. These methods just claim their respective rationality only from a certain geometric reason, and neglect almost every condition given in the mathematical formulation. For example, those never consider capacity constraint of each depot and the handling cost and the practical transportation cost accounting in the objective function. Against this, since the above auxiliary problem considers all these key conditions that are involved in the generic mathematical formulation, we can assert its rationality more relevantly.
3.2 Weber basis saving method

Saving method is a popularly known heuristic method for solving the generic VRP. Thereat, saving value that is the reward from merging the redundant paths plays a key role to drive the algorithm. Letting the suffix be 0 for depot and \( s_0 = 0 \), it is given only on distance (kilo-meter) basis as follows.

\[
s_{ij} = d_{io} + d_{oj} - d_{ij}, \quad i, j = 1, 2, \ldots, |K|, i \neq j
\]

(6)

If we will not pay attention to the special conditions on forward and backward paths, the above conventional saving values becomes always same regardless of the problem type, i.e., either delivery or pickup or SPD. Against this, it is not true for the case when we consider both distance and weight or take Weber basis. In this case, we also need to account the unladen weight of vehicle \( v, w_c \). After all, these results are summarized in Table 1. The remained parts for 3-dimensional problems are left for the readers since it is easy to derive each by drawing a similar scheme shown in Fig.10.

<table>
<thead>
<tr>
<th>Type</th>
<th>Weber model ((s_0/c_v))</th>
<th>Generalized Weber model ((s_0'/q(c_v)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost accounting btw (i) and (j)</td>
<td>Bilinear: (c_v g_d d_{ij})</td>
<td>Power: (c_v</td>
</tr>
<tr>
<td>Delivery</td>
<td>( q_i(d_{io} - d_{io} - d_{ij}) + w_i(d_{io} + d_{io} - d_{ij}) )</td>
<td>((w_i + q_j)^\alpha - (w_i + q_i + q_j)^\alpha) (d_{ij}^\beta)</td>
</tr>
<tr>
<td>Pick up</td>
<td>( p_i(d_{io} - d_{io} - d_{ij}) + w_i(d_{io} + d_{io} - d_{ij}) )</td>
<td>((w_i + p_j)^\alpha - (w_i + p_i + p_j)^\alpha) (d_{ij}^\beta)</td>
</tr>
<tr>
<td>Drop by</td>
<td>((p_i + w_i)(d_{io} - d_{io} - d_{ij}) + w_i(d_{io} + d_{io} - d_{ij}) - p_id_{ij})</td>
<td>((w_i + p_j)^\alpha - (w_i + p_i + p_j)^\alpha) (d_{ij}^\beta)</td>
</tr>
<tr>
<td>SPD</td>
<td>(d_{io}(p_i - q_j + w_c) + d_{io}(-p_i + q_j + w_c))</td>
<td>((w_i + q_j)^\alpha G_0 d_{ij}^\beta + (w_i + p_i)^\alpha G_0 d_{ij}^\beta + (w_i + q_j)^\alpha G_{ij} d_{ij}^\beta)</td>
</tr>
<tr>
<td>3- dimension</td>
<td>Delivery</td>
<td>(q_i(G(z_{io})d_{io} - G(z_{io})d_{io} - G(z_{io})d_{ij}) + w_i(G(z_{io})d_{io} + G(z_{io})d_{io} - G(z_{io})d_{ij}))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Various saving values \(s_{ij}\).

![Fig.10 Scheme available for deriving Weber saving value for 3-dimensional transportation (delivery); Round path is merged if we can save the value in terms of the extended Weber basis accounting.](image)

Though the algorithm of the proposed Weber basis saving method is basically same as the original one, the proposed one takes the fixed-charge of vehicle \( C_{fix} \) into account besides the routing cost. This procedure is outlined as follows.

Step 1: Create round trip routes from the depot for every pair, and compute the Weber basis savings value.
Step 2: Order these pairs in descending order of such savings value.
Step 3: Merge the path in turn following the order obtained from Step 2 as long as it is feasible and the savings
value is greater than \(-F_v/c_v\), where \(F_v\) denotes the fixed operational cost of vehicle \(v\).

The above Step 3 modifies the original idea in terms of such assertion that visiting the new customer is more economical even if its saving cost would become negative as long as its absolute value stays within the fixed operational cost of the additional vehicle. Including such fixed charge and weight of unladen vehicle in evaluations are our original ideas. Through this method, we can derive the initial routes more practically and consistently compared with the conventional one. Finally, we can evaluate the total transportation cost \(TC\) by Eq.(7).

\[
TC = \sum_{i=1}^{L} TR_i + L_R F_v
\]

where \(TR_i\) denotes routing cost of root \(i\), and \(L_R\) total number of routes (necessary vehicle number).

3.3 Modified tabu search

Since the Weber basis saving method derives only an approximated solution, we try to improve it by applying the modified tabu search. The tabu search is a simple but powerful heuristic method that refers to a local search with certain memory structure. In its local search applied in the inner loop, we generate a neighbour solution from either of insert, swap or 2-opt operations within the route and either of insert, swap or cross operations between the routes by randomly selecting every candidate. On the other hand, in the outer loop search, an extended swap (Shimizu, Sakaguchi & Yoo, 2016) is used to generate a neighbour solution. To avoid trapping into a local minimum, our modified method allows even a degraded neighbour solution to be a new tentative solution as long as it would be feasible and not be involved in the tabu list. Such decision is made in terms of the probability whose distribution obeys the following Maxwell-Boltzmann function and used in simulated annealing.

4. Numerical experiment

We prepared benchmark problems for numerical experiments as follows. We randomly generated the prescribed numbers of customers within a rectangular region. On the other hand, depot is placed at the centre for single-depot problem while they are distributed randomly within the smaller region involved in the entire region for multi-depot problem. The distances between depot and customers for multi-depot problem and also between every customer are given by Euclidian basis. For 3-dimensional problems, elevation data as the weighted average factor defined in Sec.2.5 are randomly given within a certain prescribed range. Likewise, each demand of customer is randomly given within a certain prescribed range.

Moreover, for the generalized problems, we set \(\alpha=0.894, \beta=0.750, \gamma=1.726\) referring to the literature mentioned before. Size of tabu list and number of inner and outer loop iterations are changed depending on the problem size. We used PC with CPU: Intel(R) Core(TM)2 Quad Processor Q6600 2.4GHz, and RAM: 3GB. The following discussions are made based on the results averaged over 10 runs per each problem.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Type</th>
<th>Size*1</th>
<th>Bilinear Gain</th>
<th>CPU [s]</th>
<th>Gain</th>
<th>CPU [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mono</td>
<td>Single-Deliver [1]</td>
<td>[1, 1000]</td>
<td>0.285</td>
<td>38.1</td>
<td>0.149</td>
<td>207.2</td>
</tr>
<tr>
<td></td>
<td>Single-Pick(Direct)</td>
<td>[1, 1000]</td>
<td>0.290</td>
<td>40.1</td>
<td>0.113</td>
<td>210.2</td>
</tr>
<tr>
<td></td>
<td>Single-Pick(Drop-by)</td>
<td>[1, 1000]</td>
<td>0.214</td>
<td>41.5</td>
<td>0.111</td>
<td>208.5</td>
</tr>
<tr>
<td></td>
<td>Multi-Deliver [1]</td>
<td>[10, 100]</td>
<td>0.037</td>
<td>230.1</td>
<td>0.046</td>
<td>977.4</td>
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<tr>
<td></td>
<td>Multi-Pick(Direct)</td>
<td>[10, 100]</td>
<td>0.041</td>
<td>277.2</td>
<td>0.035</td>
<td>1701.4</td>
</tr>
<tr>
<td>Multi</td>
<td>Single-SPD [2]</td>
<td>[1, 1000]</td>
<td>0.804</td>
<td>133.9</td>
<td>0.611</td>
<td>289.7</td>
</tr>
<tr>
<td></td>
<td>Multi-SPD [3]</td>
<td>[10, 100]</td>
<td>0.621</td>
<td>309.2</td>
<td>0.601</td>
<td>1986.6</td>
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</tbody>
</table>


In Tables 2, we summarized a part of the results solved for all variants in the previous studies to carry out the overall evaluation (Regarding the more detailed discussion for those results, refer to each study cited below in the table). First, we know it possible to solve all within acceptable computation times even for such large problems that have never been solved elsewhere. Here, ‘Gain’ denotes the improved rate of the final solution from the initial one (result from the Weber basis saving method). It is equivalent to ‘Rate’ and ‘Rate2’ in the previous studies for single and
multi-depot problems, respectively. In other words, it stands for the gain of the modern heuristics developed in our framework over the classical heuristics like saving method.

From these, we can claim as follows:

1. Considerable updates are done by the proposed approach except for the ‘mono-mode multi-depot’ problems.
2. Multi-mode case (SPD) gets larger improvement compared with the mono-mode. In turn, it might suggest the poor performance of Weber basis saving method for the SPD problems. However, its poor performance is recovered by the modified tubu search.

Regarding what happened in the above items 1 and 2, we discussed on the reasons in our previous studies. Moreover, we numerically confirmed the advantage of decentralized over the centralized and that of multi-mode (SPD) over separate mono-mode as the special interest mentioned in Sec.2.2 and 2.3, respectively.

Next, we show the result of the 3-dimensional case in Table 3. We set the parameters of sigmoid function mentioned in Sec.2.5 as $a=0.02$, $b=2.0$ and $c=25$ for the upward and $a=0.45$, $b=1.0$ and $c=30$ for the downward. Having solved only the single-depot delivery problem, we know its performance is pretty good and equivalent to the 2-dimensional case as a whole. Moreover, as shown in Fig. 11, we can confirm the sufficient convergence both for the ordinary and the generalized Weber model with 1000 customers. These facts still claim the high performance of the proposed approach.

Table 3 Results of 3-dimensional problem (Mono-mode single delivery)

<table>
<thead>
<tr>
<th>Size</th>
<th>Bilinear Gain</th>
<th>CPU [s]</th>
<th>Power Gain</th>
<th>CPU [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.230</td>
<td>2.4</td>
<td>0.222</td>
<td>5.0</td>
</tr>
<tr>
<td>200</td>
<td>0.179</td>
<td>12.0</td>
<td>0.183</td>
<td>22.6</td>
</tr>
<tr>
<td>300</td>
<td>0.263</td>
<td>32.5</td>
<td>0.181</td>
<td>64.8</td>
</tr>
<tr>
<td>500</td>
<td>0.228</td>
<td>199.8</td>
<td>0.205</td>
<td>202.7</td>
</tr>
<tr>
<td>1000</td>
<td>0.302</td>
<td>514.5</td>
<td>0.243</td>
<td>1038.6</td>
</tr>
</tbody>
</table>

Fig. 11 Profile of convergence for 3-D problem; Sufficient convergence might guarantee the effectiveness of the proposed approach numerically.

Figure 12 illustrates the profiles of the gap defined in Sec.2.5 both for Weber and its generalized bases. Definition of the gap is shown again as a scheme in the right hand side of Fig.12. Since every value takes positive value, we know it possible to reduce the actual cost by additionally using the elevation date. As it were, it represents a rate of opportunity loss due to missing available data like elevation in optimization. Moreover, its magnitude is pretty large and increases along with the problem size. Thus, considering the elevation has a great advantage over the conventional dealing that has ignored it and promises a certain deployment towards innovative logistics in future real-world applications.

In a summary, we claim the Weber basis saving method generally behaves well for mono-mode problems. Its performance will be amplified by incorporating the customer allocation algorithm for multi-depot problems. From the complexity, solution performance of the generalized Weber basis problems is substantially inferior to that of Weber basis problems, e.g., lower gains and longer CPU times. Moreover, so called “curse of dimensionality” may not be a fatal difficulty when solving large problems. Actually, in the above computation environment, we could solve the 2-dimensional problems with size up to [25, 3000] in acceptable computation times. Finally, we emphasize again such
flexibility of our approach that can handle every variant concerned here in the common framework. As a nature of evolutionary method, however, it is unable to guarantee the optimality of the solutions obtained here. Moreover, we cannot compare the performance with other methods since any methods that adopt the Weber basis and/or 3-dimensional mode has never been known elsewhere. Then, through a series of numerical experiments shown in the above and various analyses done together, we have shown a well-approximated solution is surely derived by the proposed method within an acceptable computation time even for larger problems. This fact is of special importance for many real world applications.

Fig. 12 Profiles of the gap between the costs evaluated on 2- and 3-dimensional bases; Considering the elevation provides more realistic environment (3D) than the conventional convenient one (2D). Hence that can derive more relevant solution (3D*) compared with one (2D*) ignoring that. The gap evaluates the relative cost difference between those solutions in real environment, i.e. 3D.

5. Conclusion

As a key technology for logistics optimization under global manufacturing and demand on qualified service, this study considers a general and practical framework of the algorithm for VRP under various interests. Then, its scope is enhanced by adding the idea on the 3-dimensional transportation. Thereat, making the best use of the graph algorithm of MCF problem, the Weber basis saving method and the modified tabu search in a hierarchical manner, we have provided a procedure effective to cope with various real world applications.

We discussed on the numerical experiments performed so far to comprehensively evaluate the effectiveness of the proposed approach. Including the inspection on the results of 3-dimensional delivery problem, we claim the great possibility and practice of our framework. In fact, it employs a practical Weber basis cost accounting involving the effect of own weight of vehicle. Depending on the available information or the decision environment, it is easy to extend to more practical power model and/or 3-dimensional formulation. Moreover, it has a flexibility to straightforwardly import some new findings on heuristics in local search to improve the solution ability and cope with other variants.

In future studies, we aim at turning our interests toward general multi-objective optimizations to promote sustainability and safety in modern global and dynamic logistic systems. It is interesting to study the 3-dimensional logistic studies associated with Elevation API of Google map to retrieve real geographic data and illustrate the numerical result on the map.

References


